wjec cbac

GCE AS/A LEVEL – NEW

2305U30-1

S19-2305U30-1

MONDAY, 20 MAY 2019 – AFTERNOON

FURTHER MATHEMATICS – AS unit 3
FURTHER MECHANICS A

1 hour 30 minutes

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

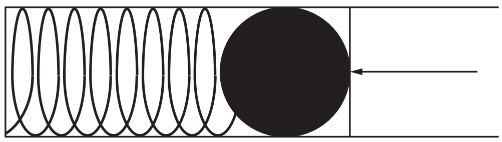
Use black ink or black ball-point pen.
Answer **all** questions.
Take g as $9{\cdot}8\text{ms}^{-2}$.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Answers without working may not gain full credit.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: *Sufficient working must be shown to demonstrate the* ***mathematical*** *method employed.*

1. The diagram shows a spring of natural length 0·15 m enclosed in a smooth horizontal tube. One end of the spring *A* is fixed and the other end *B* is compressed against a ball of mass 0·1 kg.



Initially, the ball is held in equilibrium by a force of 21 N so that the compressed length of the spring is $\frac{2}{5}$ of its natural length.

(a) Calculate the modulus of elasticity of the spring. [3]

(b) The ball is released by removing the force. Determine the speed of the ball when the spring reaches its natural length. Give your answer correct to two significant figures. [5]

2. A particle of mass 0·5 kg is moving under the action of a single force $\mathbf{F}$ N so that its velocity $\mathbf{v}$ ms^{-1} at time t seconds is given by

$$\mathbf{v} = 3t^2\mathbf{i} - 8t\mathbf{j} + 2e^{-t}\mathbf{k}.$$

(a) Find an expression for the acceleration of the particle at time t s. [2]

(b) Determine an expression for $\mathbf{F.v}$ at time t s. [3]

(c) Find the kinetic energy of the particle at time t s. [3]

(d) Describe the relationship between the kinetic energy of a particle and the rate of working of the force acting on the particle. Verify this relationship using your answers to part *(b)* and part *(c)*. [2]

3. The position vectors $\mathbf{r}_A$ and $\mathbf{r}_B$, in kilometres, of two small aeroplanes *A* and *B* relative to a fixed point *O* are given by

$$\mathbf{r}_A = (60\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}) + (168\mathbf{i} + 132\mathbf{j})t,$$

$$\mathbf{r}_B = (62\mathbf{i} + 3\mathbf{k}) + (160\mathbf{i} + p\mathbf{j} + q\mathbf{k})t,$$

where t denotes the time in hours after 9:00 a.m. and p, q are constants.

The aeroplanes *A* and *B* are on course to collide.

(a) Show that p = 140 and q = 4. [4]

(b) Find an expression for the square of the distance between *A* and *B* at time t hours after 9:00 a.m. [2]

(c) Both aeroplanes have systems that activate an alarm if they come within 600 m of each other. Determine the time when the alarms are first activated. [4]

4. A car of mass 1200 kg has an engine that is capable of producing a maximum power of 80 kW. When in motion, the car experiences a constant resistive force of 2000 N.

(a) Calculate the maximum possible speed of the car when travelling on a straight horizontal road. [3]

(b) The car travels up a slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{20}$.
If the car's engine is working at 80% capacity, calculate the acceleration of the car at the instant when its speed is 20 ms^{-1}. [5]

(c) Explain why the assumption of a constant resistive force may be unrealistic. [1]

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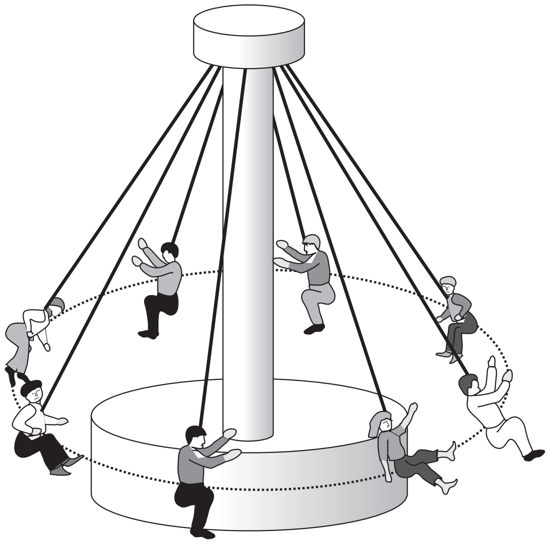
TURN OVER

5. The diagram shows a fairground ride that consists of a number of seats suspended by chains that swing out as the centre rotates.



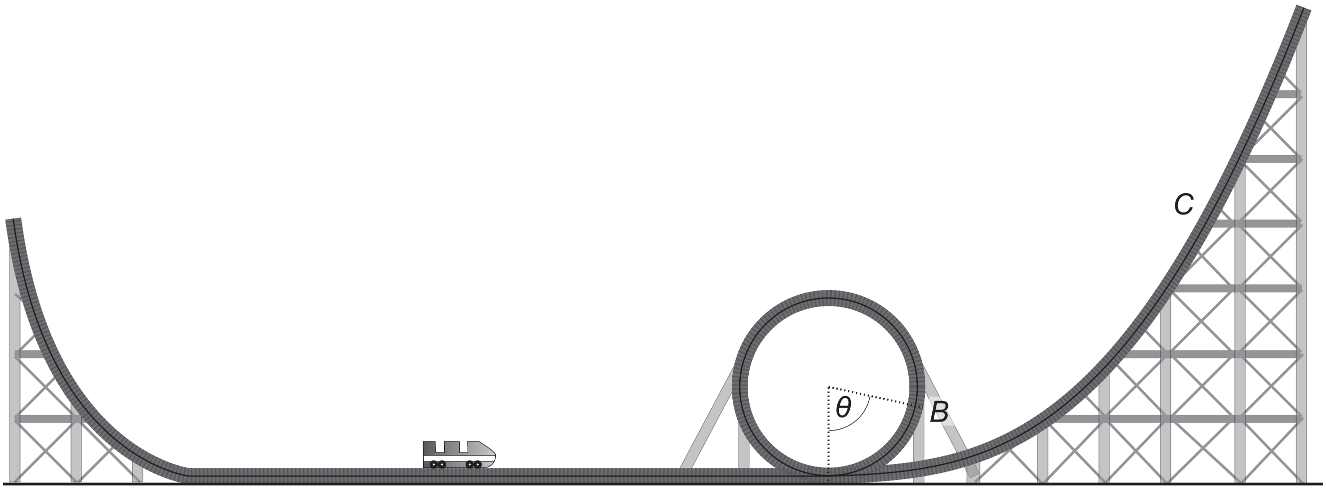
When the ride rotates at a constant angular speed of ω = 1·4 rad s^{-1}, the seats move in a horizontal circle with each chain making an angle θ with the vertical. Each of the seats and the chains may be modelled as light. Assume that all chains have the same length and are inextensible.

When a man of mass 75 kg occupies a seat, the tension in the chain is $490\sqrt{3}$ N.

(a) Show that θ = 30°. [3]

(b) Calculate the length of each chain. [5]

6. The diagram shows a rollercoaster at an amusement park where a car is projected from a launch point *O* so that it performs a loop before instantaneously coming to rest at point *C*. The car then performs the same journey in reverse.



The loop section is modelled by considering the track to be a vertical circle of radius 10 m and the car as a particle of mass m kg moving on the inside surface of the circular loop. You may assume that the track is smooth.

At point *A*, which is the lowest point of the circle, the car has velocity u ms^{-1} such that $u^2 = 60g$. When the car is at point *B* the radius makes an angle θ with the downward vertical.

(a) Find, in terms of θ and g, an expression for v^2, where v ms^{-1} is the speed of the car at *B*. [4]

(b) Show that R N, the reaction of the track on the car at *B*, is given by

$$R = mg(4 + 3\cos\theta).$$ [4]

(c) Explain why the expression for R in part *(b)* shows that the car will perform a complete loop. [2]

(d) This model predicts that the car will stop at *C* at a vertical height of 30 m above *A*. However, after the car has completed the loop, the track becomes rough and the car only reaches a point *D* at a vertical height of 28 m above *A*. The resistance to motion of the car beyond the loop is of constant magnitude $\frac{mg}{32}$ N. Calculate the length of the rough track between *A* and *D*. [3]

TURN OVER

7. Three spheres *A*, *B*, *C*, of equal radii and each of mass m kg, lie at rest on a smooth horizontal surface such that their centres are in a straight line with *B* between *A* and *C*. The coefficient of restitution between *A* and *B* is e. Sphere *A* is projected towards *B* with speed u ms^{-1} so that it collides with *B*.

(a) Find expressions, in terms of e and u, for the speed of *A* and the speed of *B* after they collide. [7]

You are now given that $e = \frac{1}{2}$.

(b) Find, in terms of m and u, the loss in kinetic energy due to the collision between *A* and *B*. [2]

(c) After the collision between *A* and *B*, sphere *B* then collides with *C*. The coefficient of restitution between *B* and *C* is e_1. Show that there will be no further collisions if $e_1 \leqslant \frac{1}{3}$. [3]

END OF PAPER

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